TDA Progress Report 42-95 July-September 1988

# A Preliminary Weather Model for Optical Communications Through the Atmosphere

K. S. Shaik Communications Systems Research Section

A preliminary weather model is presented for optical propagation through the atmosphere. It can be used to compute the attenuation loss due to the atmosphere for desired link availability statistics. The quantitative results that can be obtained from this model provide good estimates for the atmospheric link budget necessary for the design of an optical communication system. The result is extended to provide for the computation of joint attenuation probability for n sites with uncorrelated weather patterns.

#### I. Introduction

During the past few decades, the pioneering work of Tatarski [1], Fried [2,3], Hufnagel and Stanley [4], and Ishimaru [5], has set the stage for the development of a consistent theory to evaluate the performance of optical communications and imaging through the atmosphere. However, there is a conspicuous lack of weather models that may be used to obtain even a first-order estimate of statistics for the availability of an optical communications link under ambient weather conditions.

The light energy of a beam dissipates as it travels through the atmosphere, due to scattering and absorption. If it can be assumed that (1) attenuation loss is independent of radiation intensity and (2) the absorbing and scattering events occur independently of each other, the atmospheric attenuation can be expressed by the Bouguer-Lambert law:

$$I(v) = I_0(v) \exp \left[ -\int_0^Z \gamma(v, z) \, dz \right]$$
 (1)

where  $I(\nu)$  is the observed irradiance at optical frequency  $\nu$ ,  $I_0(\nu)$  is the irradiance that would have been observed if the beam were propagating in a vacuum at a distance Z from the source, and  $\gamma(\nu, z)$  is the total extinction coefficient due to scattering and absorption by the atmospheric constituents at position z. The magnitude of the argument of the exponential in the above equation is known as the optical depth or optical thickness,  $\tau$ , of the medium, i.e.,

$$\tau = \int_0^Z \gamma(\nu, z) dz \tag{2}$$

Experiments [6] with artificial fog and smoke and diluted solutions of milk show that the Bouguer-Lambert law holds well for optical thickness  $\tau \leq 12$ . The optical depth can be simply related to the attenuation loss, L, in dB by the following approximation:

$$L \approx 4.34\tau \tag{3}$$

The distribution of gas molecules, clouds, fog, haze, aerosols, and other particulates in the transmission path influence the computation of attenuation loss. These phenomena at best can be interpreted on a statistical basis. The system designer may have to be content with a probability for which the attenuation of the atmosphere due to ambient weather is less than some critical value L dB. Hence we define the probability,  $w_1(L)$ , of the random weather variable, W, for a single site as

$$w_{1}(L) = P_{W}(l \leq L) \tag{4}$$

where l is the loss variable.  $w_1(L)$  is the fraction of weather conditions at any given site for which the attenuated direct beam irradiance  $I(\nu) \ge I_0(\nu) \exp[-0.23 \ L]$ . The weather model developed below can be used to compute this probability. The result is also extended to obtain the joint weather probability,  $w_n(L)$ , that at least one of the sites has extinction loss  $l \le L$  for n independent sites receiving simultaneously.

#### II. Weather Model

The following observations are made in order to arrive at the preliminary weather model:

- (1) It is assumed that for some fraction of the time p, where  $0 \le p \le 1$ , clear weather conditions hold. This number can be determined approximately from existing data on cloud cover and visibility for potential sites. An analysis of two years of GOES satellite data by Wylie and Menzel [7] shows that the probability of having clear weather in the southwestern U.S. is over 60% (Fig. 1).
- (2) The attenuation under clear weather conditions is due to scattering and absorption by air molecules and sparse particulate matter in the upper atmosphere. The attenuation due to molecular absorption is approximately 0.5 dB [6]. The attenuation due to molecular scattering is of the same order. Given that the atmosphere is never totally free of aerosols and thin cirrus clouds, on an average clear day the attenuation loss would be in the range of 1 to 3 dB. For simplicity, let us assume that the minimum attenuation loss due to the atmosphere is 3 dB. Hence, the model defines clear-air atmosphere, which occurs with probability p, as having an attenuation loss  $L_0 = 3$  dB. Note that this attenuation loss refers to near-zenith propagation through the entire atmosphere. For any zenith angle  $\theta$ , the path attenuation can be written as  $L_0$  sec  $\theta$ . However, the remainder of the discussion assumes nearzenith propagation paths, i.e.,  $\theta = 0$ .
- (3) The probability of not having clear weather is q = 1 p. The attenuation loss of the atmosphere increases

rapidly with increasing concentration of aerosols, fog, haze, and clouds. Optical attenuations  $L>1000~\mathrm{dB}$  are possible, where the validity of the Bouguer-Lambert law is questionable. Table 1 shows typical ranges of attenuation values for various types of clouds [8]. Since it is unlikely that an optical communication system will be designed for  $L>30~\mathrm{dB}$ , the Bouguer-Lambert law serves as a good approximation. As shown in Fig. 2, the probability of opaque cloud cover over the southwestern U.S. is 20% [7]. Let us take a pessimistic view and assume that attenuation loss due to all opaque clouds is  $100~\mathrm{dB}$  or higher, and use this assumption to estimate one of the model parameters.

(4) It is further assumed that the attenuation of the beam has an exponential distribution for  $L \ge L_0$ . The hypothesis is supported by visual observations, and also by the experimental data collected at Goldstone in Calfornia for atmospheric propagation at 8 to 10 Ghz.<sup>1</sup>

With the foregoing assumptions in mind, it is now possible to postulate a weather model. The weather cumulative distribution function (CDF) for a single site can be modeled as

$$w_1(L) = 1 - q \exp \left[-0.23 b(L - L_0)\right] \qquad (L \ge L_0)$$
(5)

where  $w_1(L)$  is defined in Eq. (3) above, q is the probability fraction when the weather is not clear, b is a site-dependent parameter to model the slope of the CDF curve, and  $L_0$  is the minimum attenuation loss due to the atmosphere. Note that  $w_1(L=L_0)=p$ . The value of b may vary with geographical location and altitude, and can be inferred from observed visibility and extinction loss data for potential receiving sites. For the southwestern U.S. region, q=0.4, and from the third item above, the attenuation loss is  $100 \, \mathrm{dB}$  when  $w_1(L=100)=0.8$ . Using these values in Eq. (5), we find that b=0.03. This estimate of the parameter b will be used in numerical examples later.

Equation (5) can be recast in a more familiar form for an optical communication system designer to give the attenuation loss in terms of the weather probability  $w_1(L)$ , i.e.,

$$L = \begin{cases} L_0, & \text{for } w_1(L) \leq p \\ L_0 + \frac{1}{0.23 \, b} \, \ln \left[ \frac{q}{1 - w_1(L)} \right], & \text{for } w_1(L) > p \end{cases}$$
(6)

<sup>&</sup>lt;sup>1</sup> JPL Internal Document 810-5, Rev. D, 1988.

Figure 3 is a plot of attenuation loss as a function of the weather probability for the southwestern region of the U.S., for which q = 0.4, b = 0.03, and  $L_0 = 3$  dB. These estimates, needless to say, are quite important for the system designer to determine the link budget for loss due to the atmosphere for Earth-space optical communication paths.

It is also possible to extend the result to n sites receiving simultaneously. From Eq. (4), it is easy to see that the complement of the probability  $w_1(L)$  is given by

$$P_{W}(l > L) = w_{1}^{c}(L) = q \exp[-0.23 b (L - L_{0})]$$

$$(L \ge L_{0}) \qquad (7)$$

The joint probability of n sites,  $w_n^c(L)$ , that the attenuation loss  $l_i > L$  for all i can be written as

$$w_n^c(L) = P_{w_1, w_2, \dots, w_n}(l_1 > L, l_2 > L, \dots, l_n > L)$$
(8)

where the subscripts  $1, 2, \ldots, n$  label the receiving sites. When the weather conditions for all sites are independent and identically distributed (IID), we have

$$w_n^c(L) = P_{W_1}(l_1 > L) P_{W_2}(l_2 > L) \dots P_{W_n}(l_n > L)$$

$$= [P_W(l > L)]^n$$
(9)

Using Eq. (9), the joint probability that at least one site has attenuation  $l \le L$  is found to be

$$w_n(L) = 1 - w_n^c(L) = 1 - [q \exp[-0.23 b (L - L_0)]]^n$$

$$(L \ge L_0) \tag{10}$$

For a single site with p = 0.6,  $L_0 = 3$  dB, and b = 0.03, the probability that the attenuation  $L \le 3$  dB is  $w_1(L = 3) = 0.60$ . If three such IID sites are chosen, we have  $w_3(L=3)=0.94$ . In other words, if a system is designed to absorb an extinction loss of 3 dB, a three-site receiving network will be functional 94% of the time. Figure 4 plots the fraction of the total period under ambient weather conditions when the attenuation is ≤ 3 dB as a function of the number of sites. Table 2 gives the expected dB loss for a desired link availability percentage for up to four joint receiving sites. It is, however, not very clear how the independence of weather patterns at various sites can be insured. It is known that the scale size of weather patterns is on the order of a few hundred kilometers, and this measure may be used to find sites with uncorrelated weather. Joint observation of weather parameters for the probable sites will be necessary to make a more accurate determination.

#### III. Conclusion

The virtue of the weather model presented here lies in its simplicity. It may be applied with ease to obtain a first-order magnitude of probabilities  $w_n(L)$  for an optical system that must operate in the atmosphere. The computation of these probabilities, for example, will provide a good statistical estimate of the attenuation loss to an optical communication system designer for link availability.

The model does not consider frequency dependence, since it has been studied thoroughly and well documented in LOW-TRAN computer code developed at the Air Force Geophysics Laboratory [9]. The model also disregards seasonal variations, which can be incorporated later when adequate data bases have been developed.

## **Acknowledgment**

The author wishes to thank James K. Lesh for his comments and discussions, and also Dr. Chien-Chu Chen for his helpful suggestions.

### References

- [1] V. I. Tatarskii, Wave Propagation in a Turbulent Medium, New York: Dover, 1967.
- [2] D. L. Fried, "Propagation of a Spherical Wave in a Turbulent Medium," J. Opt. Soc. Am., vol. 57, pp. 175-180, 1967.
- [3] D. L. Fried, "Limiting Resolution Looking Down Through the Atmosphere," J. Opt. Soc. Am., vol. 56, pp. 1380-1384, 1966.
- [4] R. E. Hufnagel and N. R. Stanley, "Modulation Transfer Function With Image Transmission Through Turbulent Media," J. Opt. Soc. Am., vol. 54, pp. 52-61, 1964.
- [5] A. Ishimaru, Wave Propagation and Scattering in Random Media, New York: Academic Press, 1977.
- [6] V. E. Zuev, Laser Beams in the Atmosphere, New York: Consultants Bureau, 1982.
- [7] D. P. Wylie and W. P. Menzel, "Cloud Cover Statistics Using VAS," SPIE's OE-LASE'88 Symposium on Innovative Science and Technology, Los Angeles, CA, January 10-15, 1988.
- [8] The Technical Cooperation Program, Volume V—Laser Communications Workshop held in Australia, U.S. Government Printing Office no. 559-065/20988, 1985.
- [9] F. X. Kneizys, et al., "Atmospheric Transmittance/Radiance: Computer Code LOW-TRAN6," Rep. AFGL-TR-83-0187, 1983.

Table 1. Typical range of values for cloud attenuation (adapted from [8])

Cloud type	Cloud base height, km	Cloud thickness, km	Total vertical attenuation, dB	
Stratus	0.1 - 0.7	0.2 - 0.8		
Stratocumulus	0.6 - 1.5	0.2 - 0.8	3 - 100	
Nimbostratus	0.1 - 1.0	2 - 3	260 - 1300	
Altostratus/ cumulus	2 – 6	0.2 - 2	9 - 260	
Cumulus	0.5 - 1.0	0.5 - 5	22 - 870	
Cumulonimbus	0.5 - 1.0	2 - 12	260 - 5200	
Cirriform (ice)	6 - 10	1.0 - 2.5	1 - 15	
Fog	0	0 - 0.15	0 - 13	

Table 2. Attenuation loss as a function of desired link availability for *n* sites receiving jointly

Percentage weather	Attenuation loss, $L$ , dB				
	n = 1	n = 2	n = 3	n = 4	
60.0	3.00	3.00	3.00	3.00	
62.0	10.34	3.00	3.00	3.00	
64.0	18.07	3.00	3.00	3.00	
66.0	26.25	3.00	3.00	3.00	
68.0	34.92	3.00	3.00	3.00	
70.0	44.16	3.00	3.00	3.00	
72.0	54.03	3.00	3.00	3.00	
74.0	64.63	3.00	3.00	3.00	
7 <b>6</b> .0	76.08	3.00	3.00	3.00	
78.0	88.53	3.00	3.00	3.00	
80.0	102.16	3.00	3.00	3.00	
82.0	117.23	3.00	3.00	3.00	
84.0	134.09	3.00	3.00	3.00	
86.0	153.19	12.55	3.00	3.00	
88.0	175.24	23.58	3.00	3.00	
90.0	201.32	36.62	3.00	3.00	
92.0	233.25	52.58	3.00	3.00	
94.0	274.40	73.16	6.08	3.00	
96.0	332.41	102.16	25.41	3.00	
98.0	431.57	151.74	58.47	11.83	
99.0	530.72	201.32	91.52	36.62	

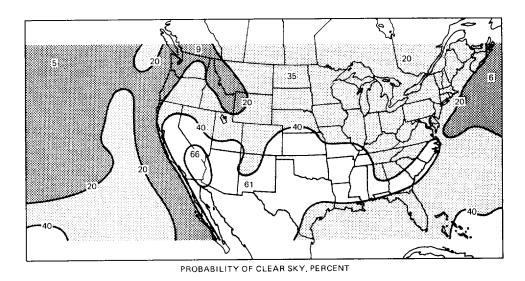


Fig. 1. Contour diagram obtained from 2 years of GOES satellite data showing the probability of clear sky over the United States [7].

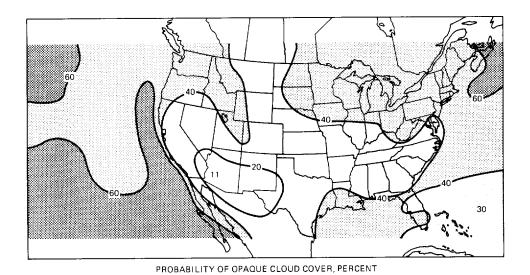


Fig. 2. Contour diagram obtained from 2 years of GOES satellite data showing the probability of opaque cloud cover over the United States [7].

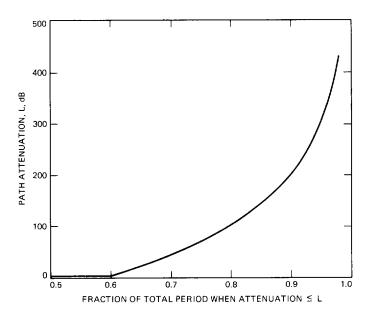


Fig. 3. Model curve for the path attenuation as a function of weather probability for a single site. Choice of parameters,  $L_0=3$  dB, b=0.03, and  $\rho=0.6$  represents estimates for the southwestern U.S.

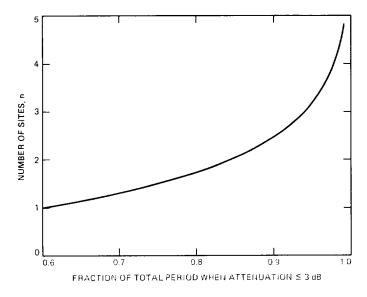


Fig. 4. Weather probability as a function of the number of joint receiving sites.  $L_0=3$  dB, b=0.03, p=0.6, and L=3 dB.